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Dynamic Characteristics of Perforated Distillation Plates Operating at Low Loads

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Analysis of the equations describing the instantaneous vapor and liquid flow through the holes of perforated distillation plates at low loads shows that periodic and stable pressure oscillations will always be set up between plates. These oscillations are expected to have amplitudes of the order of 0.1 in. of water and frequencies of a few cycles per second, values which are in accord with observation and which may be used as the basis for a model from which the seal point of the plates may be predicted.

In a recent paper (*11*) the authors have discussed the prediction of the "seal point" of perforated distillation plates (such as the plate illustrated in Figures 1 and 2 of reference *11*) which they take as corresponding to the minimum load of these plates. They have defined the seal point as the vapor rate required to just maintain the liquid level on a plate at the weir height, for a given rate of liquid supplied to the plate. This point is then to be distinguished from the "weep point," generally defined as the vapor rate required to prevent any liquid from flowing ("weeping") through the plate perforations.

At the seal point all liquid fed to the plate weeps through the perforations. An increase in vapor rate leads to a decreasing proportion of the total liquid flow weeping (until the weep point is attained), while a decrease in the vapor rate will lead to a rapid decrease in liquid height on the plate, that is, the liquid seal on the plate will be lost. The points may be identified on a plot of vapor pressure drop against vapor rate for a fixed liquid rate (or fixed ratio, liquid:vapor) (a typical plot is shown in Figure 1). At *A* no liquid seal exists on the plate; *B* corresponds to the seal point and a higher vapor rate; *C* is the weep point.

The region between the seal point and the weep point may be referred to as the "weeping range." The overall pressure drop shows little or no increase over this range. Beyond the weep point, however, each hole must carry an increasing vapor load; hence the pressure drop will increase more rapidly. In the weeping range, the overall mass transfer efficiency will be reasonable, unless a high proportion of liquid weeps through a plate on which the

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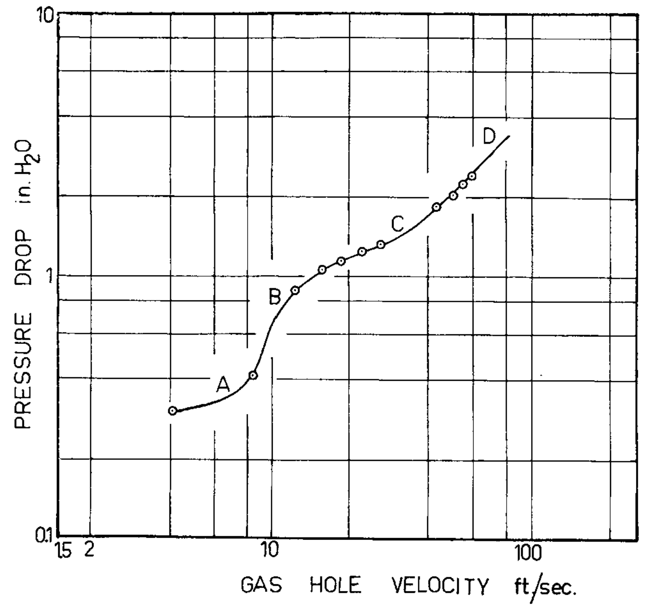

Fig. 1. Wet plate pressure drop as a function against gas hole velocity: 18 in. diameter column, 3/16 in. diameter holes, 12.5% free area, 3/4 in. weir height, air-water system. Liquid rate = 2,000 lb./(hr.)(sq.ft.).

flow approaches plug flow, but below the seal point the efficiency will drop off very rapidly, owing to the decreasing contact between liquid and vapor. The seal point then defines a minimum allowable load and its prediction is of importance.

The weeping range, of which the seal point is one limit, is characterized by both liquid flowing downward through some or all holes in the plate and vapor flowing upward, again through some or all holes, under a constant set of external conditions. In the paper referred to (*11*) a model was set up for fluid flow under these conditions by postulating that the variation of liquid to vapor flow through a particular hole is caused by periodic pressure fluctuations. By further postulating that the amplitude of these fluctuations is the minimum required to obtain the (fixed) overall fluid flow through the plate, the seal point corresponding to any set of external conditions could be predicted.

It is intended in this paper to show, by considering the instantaneous flow of liquid or vapor through the plate perforations, that pressure fluctuations will always be set up in a multiplate column, that they are stable, and of a type and magnitude required by the above postulates and consistent with observation.

PREVIOUS WORK

Brown (*1*), investigating the pressure in the space below a single perforated plate, found that it fluctuated in a periodic manner. By using various gas-liquid systems, hole diameters of ¼ to ½ in., and chamber volumes of 200 to 300 cc., he measured the amplified changes of strain on a thin metal diaphragm tightly stretched across a 2-in. opening at the side of the chamber below the plate. His results are summarized in Table 1.

TABLE 1. FREQUENCIES OF PRESSURE FLUCTUATIONS (*1*)

No. of holes on plate	Mean frequency, cycles/sec.	Range of frequencies, cycles/sec.
1	9.5	3.3 to 14.3
3	11.4	7.8 to 18.3
7	18.0	11.8 to 26.0
29	23.9	23.5 to 25.0
41	27.4	26.0 to 28.5
49	27.9	24.5 to 32.0

The frequencies were correlated as

$$\frac{DF}{U_h} = 46{,}700 \left(\frac{nD^3}{V_c}\right)^{0.46} \left(\frac{D_h U_h \rho_G}{U_G}\right)^{-0.69} \left(\frac{D}{D_h}\right)^{-0.54} \left(\frac{\rho_G}{\rho_L}\right)^{0.84}$$

with a standard deviation of 10%. No general correlation for amplitudes was obtained. The pressure traces indicated that the fluctuations were about a nonzero mean value, with apparently random, but not unbounded, amplitudes, that is, the pressure trace was continuous with time.

McAllister, McGinnis, and Plank (*6*) reported an oscillating region in column operation, where the aerated liquid mass on the plate moved back and forth at right angles to the direction of liquid flow. An equation was proposed by McAllister and Plank (*7*) which related the frequency of such oscillations to the velocity of sound, the physical dimensions of the system, and the gas and liquid flow rates.

Static pressure drops are required in the equations discussed below, and while they have been reported by many authors, the correlations proposed are, in general, sufficiently accurate over only limited ranges of conditions. The most useful correlation for our purpose is due to McAllister et al. (*6*).

DYNAMIC EQUATIONS FOR THE SYSTEM

If p_1, p_2, p_3, p_4 are pressure drops due to capacitance, resistance, inertia, and any other effects, respectively, we have for a given hole through which vapor is flowing:

$$p_i = p_1 + p_2 + p_3 + p_4 \tag{1}$$

For a chamber below a plate, volume V_c, the capacitance C_o is given by

$$C_o = \frac{\text{Volume of vapor retained}}{\text{Unit pressure potential}}$$

$$= \frac{dv}{dp_1} = \frac{1}{\rho_G} \cdot \frac{dm}{dp_1} = \frac{1}{\rho_G} \cdot \frac{dm}{dV} \cdot \frac{dV}{dp_1}$$

$$= \frac{1}{\rho_G}\left(-\frac{MV_c}{V^2}\right)\left(-\frac{V}{p_1\gamma}\right),$$

since $p_1 V^\gamma = \text{constant}$

$$= \frac{V_c}{p_1\gamma}, \quad \text{since} \quad V = \frac{MV_c}{m} = \frac{M}{\rho_G}$$

$$= \frac{V_c}{c^2 \rho_G}, \quad \text{since} \quad c = \sqrt{\frac{p_1\gamma}{\rho_G}}$$

so that

$$-\frac{dp_1}{dt} = C(v_i - \bar{v}) \tag{2}$$

where

$$C = \frac{1}{C_o} = c^2 \rho_G / V_c \tag{2a}$$

if $\bar{v}$ is the time-averaged value of the volumetric vapor rate.

As the vapor passes up through the hole and the liquid layer, the resistances which it must overcome are due to contraction, friction, and expansion, as well as the static liquid head. If p_2 is taken (*6*) as

$$-p_2 = k \frac{\rho_G U_h^2}{2} \left[0.4(1.25 - \alpha) + 4f \frac{t_p}{D_h} + (1 - \alpha)^2 \right] + g_c \rho_L h_L \tag{3}$$

then

$$-\frac{dp_2}{dt} = Bv_i\frac{dv_i}{dt} \qquad (4)$$

where

$$B = \frac{16\rho_G k}{\pi^2 D_h^4}\left[0.4(1.25-\alpha) + 4f\frac{t_p}{D_h} + (1-\alpha)^2\right] \qquad (4a)$$

Owing to the inertia of the vapor in one direction and of the liquid in the opposite direction, a differential pressure p_3 is created. Consider a hole with diameter D_h, let l be a measure of the vertical distance travelled by an element of vapor in time t. The vapor velocity through the hole is $U_h = dl/dt$.
Now

$$v_i = \frac{\pi}{4}D_h^2 U_h = \frac{\pi}{4}D_h^2\frac{dl}{dt}$$

so that

$$\frac{dv_i}{dt} = \frac{\pi}{4}D_h^2\frac{d^2l}{dt^2}$$

The amount of vapor in the space above the hole is approximately

$$m_G = \frac{\pi}{4}D_h^2\,(t_p + h_L)$$

and by Newton's second law of motion

$$-p_3\cdot\frac{\pi}{4}D_h^2 = \frac{d}{dt}\,(m_G U_h) \qquad (5)$$

so that

$$-\frac{dp_3}{dt} = A\frac{d^2v_i}{dt^2} \qquad (6)$$

where

$$A = 4(t_p + h_L)\,\rho_G/\pi\,D_h^2 \qquad (6a)$$

If we consider the excess pressure required to overcome the inertia of the weeping liquid, the form of Equation (6) will remain unchanged with A replaced by $A_1 = (\rho_L/\rho_G)A$.

If losses due to other effects are taken to be constant (that is, independent of vapor velocity) and the effect of surface tension is neglected (for some discussion of this point, see *11*), we have

$$-\frac{dp_4}{dt} = 0 \qquad (7)$$

Equations (1), (2), (4), (6), and (7) may be combined to give

$$-\frac{dp_i}{dt} = A\frac{d^2v_i}{dt^2} + Bv_i\frac{dv_i}{dt} + C\,(v_i - \bar{v}) \qquad (8)$$

which is the dynamic equation of the vapor path for a given hole.

Equation (8) can be written for each of the holes on a plate. Interaction between holes will give further equations, and this family of equations governs the vapor-passing cycle of the system. Similar relationships may be derived for holes weeping liquid.

STABILITY ANALYSIS OF THE DYNAMIC EQUATIONS

Let us now analyze the dynamic characteristics of the system in terms of the dynamic equation for one hole, Equation (8). The question of stability may be studied by a perturbation method (method of small oscillations) (*12*):

Let

$$v = v_i - \bar{v}$$

$$p = p_i - \bar{p}$$

Substituting in Equation (8) one obtains

$$-\frac{dp}{dt} = A\frac{d^2v}{dt^2} + B\,(\bar{v} + v)\,\frac{dv}{dt} + Cv \qquad (9)$$

The system is stable if all the solutions of Equation (9) are bounded for $t > 0$.
Write

$$f = f(v) = \frac{B}{A}\,(\bar{v} + v) \qquad (10)$$

and

$$g = g(v) = \frac{Cv}{A} \qquad (11)$$

Both f and g are continuous, differentiable, bounded and are odd functions of v. Let $F(t) = -\frac{1}{A}\frac{dp}{dt}$, so that

$$F(t) = \ddot{v} + f\dot{v} + g \qquad (12)$$

We may regard solution of this equation as a problem in input/output analysis (*4*). The term $F(t)$ represents the input, while a solution of the right-hand side represents the output. From observed as well as Brown's results (*1*), it appears that an appropriate form of input would be a periodic function in the form of a Fourier series:

$$F(t) = \frac{1}{2}p + \sum_{n=1}^{\infty} p_n \sin\,(nwt - \beta_n) \qquad (13)$$

With $F(t) = 0$, Equation (12) becomes the McHarg equation (*9*):

$$\ddot{v} + f\dot{v} + g = 0 \qquad (14)$$

which can be written in terms of two first-order differential equations:

$$\dot{v} = u \qquad (15)$$

$$\dot{u} = -fu - g \qquad (16)$$

so that

$$du/dv = -fu - g/u \qquad (17)$$

The analysis for stable solutions of the system consists of examining the integral curves in the $u - v$ plane (*8, 9*).
Let

$$G = G(v) = \int_0^v g\,dv = \left(\frac{C}{2A}v^2\right) \qquad (18)$$

$$E = E(v, u) = \frac{1}{2}u^2 + G(v) \qquad (19)$$

then

$$dE/dt = -fu^2 \qquad (20)$$

By the principle of symmetry (*9*), a Poincarè limit cycle is established if such a curve Γ intersects the positive and negative axes of u, admitting a periodic solution of Equation (14) with respect to t. If E is considered as the total energy of the system, Equation (19) shows that it consists of a kinetic energy term 1/2 u^2 and a potential energy term G. Energy levels are given by putting E equal to a constant, and they are then single closed curves around the origin (the only singularity in the system). Equation (20) shows that E decreases with time.

In the phase plane diagram (Figure 2), starting from point $A(0, u_0)$ on the u axis, where $E_1 = E(0, u_0)$ cuts

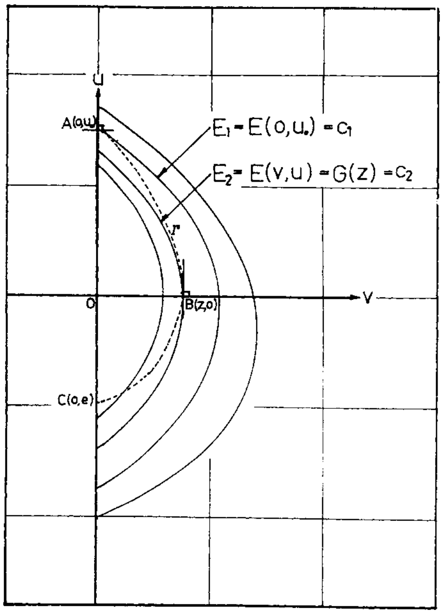

Fig. 2. Phase plane diagram.

the u axis horizontally at right angles according to Equation (20), Γ moves toward the right for $v \geqq 0$, $u > 0$. Again according to Equation (20), $dE/dt < 0$, so that Γ will lie within $E = E_0 = 1/2\ u_0^2$. Letting Γ cut Ov at $B = B(z, 0)$, we have from Equation (19) $G(z) < 1/2\ u_0^2$. At B, $u = 0$, therefore, $dE/dt = 0$, so that E is stationary at B. Hence Γ touches $E = G(z)$ at B. For $v \geqq 0$, $u \leqq 0$; since $dE/dt < 0$, on Γ we have $0 \leqq v \leqq z < v_1$ (say), and $|u| \leqq [2G(z)]^{1/2} < u_0 < \dfrac{1}{k}$ (say),

since $$\frac{1}{2}u^2 + G(v) = E \leqq G(z) < \frac{1}{2}u_0^2$$

Thus for $0 < v < v_1$, $\dfrac{du}{dv} = \left(\dfrac{g}{|v|} - f\right) > (kg - f) > 0$,

if $k > (f/g)$ for $0 < v < v_1$. Hence Γ intersects the u axis again at $C\ (0, e)$ where $-\dfrac{1}{k} < e < 0$. From B to C, $E = \left(\dfrac{1}{2}u^2 + G\right) < \dfrac{1}{2}u_0^2$, giving $u_0 \leqq 1/k$, so that u is negative, and $|u|$ is $< 1/k$. Thus Γ is a closed curve, and the solution of Equation (14) is periodic for every $u = u_0$ such that

$$0 < u_0 \leqq \min.\left\{\frac{1}{k}, [2G(v_1)]^{1/2}\right\} \tag{21}$$

Now from Equations (10) and (11), $k > f/g$ if

$$\frac{1}{k} < g/f = \frac{Cv}{B(\bar{v} + v)}$$

that is, if

$$v = v_1 < \frac{B\bar{v}}{kC - B}$$

and since

$$[2G(v_1)]^{1/2} = \sqrt{\frac{C}{A}}$$

the inequality becomes

$$0 < u_0 \leqq \min.\left\{\frac{1}{k}, \frac{B\bar{v}}{(kC - B)}\sqrt{\frac{C}{A}}\right\} \tag{22}$$

As there exists a continuous infinity of values of k satisfying this inequality for any given set of values of A, B, C, and $\bar{v}$, there can be a continuous infinity of periodic solutions for Equation (14). It can be further shown (5) that if $\int_0^t F(t)dt$ has a period τ, there exists a solution of Equation (12) of the same period. Thus there is also a continuous infinity of periodic solutions for Equation (9), and hence for the dynamic Equation (8).

ANALYSIS BY LINEARIZATION

When one considers the stable periodic solutions of the system at $\bar{v} = 0$ and writes

$$f = P = P(t) = \frac{B}{A}v \tag{23}$$

$$g = Q^2 v = \frac{C}{A}v \tag{24}$$

Equation (14) becomes

$$\ddot{v} + P\dot{v} + Q^2 v = 0 \tag{25}$$

Now P is bounded and non-negative, and Hochstadt (2, 3) has shown that periodic solutions of Equation (25) are given by

$$v = R_o \sin\phi \exp\left[-\left(\frac{B}{A}v\cos^2\phi\right)t\right] \tag{26}$$

where $R = R(t)$, $\phi = \phi(t)$, and $R = R_o$ at $t = 0$. Since $P > 0$, Equation (26) indicates that the amplitudes of the solution are bounded. For initial conditions

$$v = v_o \text{ at } t = 0$$

$$\dot{v} = 0 \text{ at } t = 0$$

and for $4Q^2 > P^2$, the solution for Equation (25) is (4)

$$v = \frac{2v_o Q\, e^{-Pt/2}}{\sqrt{4Q^2 - P^2}}\cos\left[\frac{1}{2}\ (\sqrt{(4Q^2 - P^2)}t - \tan^{-1}\frac{P}{\sqrt{4Q^2 - P^2}}\right] \tag{27}$$

that is

$$v = \frac{v_o\sqrt{4AC}\, e^{-\left(\frac{B}{2A}vt\right)}}{\sqrt{4AC - {}^2v^2}}\cos\left[\frac{1}{2A}(\sqrt{4AC - B^2v^2})t - \tan^{-1}\frac{Bv}{\sqrt{4AC - B^2v^2}}\right] \tag{28}$$

As a first approximation, the amplitude R at $t = 0$ may be taken as

$$R = \frac{v_o\sqrt{4AC}}{4AC - B^2v_o^2} \tag{29}$$

As v varies from 0 to a given value v_o, the mean amplitude is

$$\bar{R} = \frac{1}{v_o}\int_0^{v_o} R\,dv = \frac{4AC}{B^2v_o} - \frac{\sqrt{4AC}}{B^2v_o}\sqrt{4AC - B^2v_o^2} \tag{30}$$

From Equation (28) the frequency of the oscillations is given by

$$F = \frac{1}{T} = \frac{1}{4A\pi}(4AC - B^2v^2)^{1/2} \quad (31)$$

and the mean frequency $\bar{F}$ is

$$\bar{F} = \frac{1}{v_o}\int_0^{v_o} F dv =$$

$$\frac{1}{2\pi}\left[\frac{1}{4A}(4AC - B^2v_o^2)^{1/2} + \frac{C}{Bv_o}\sin^{-1}\frac{Bv_o}{\sqrt{4AC}}\right] \quad (32)$$

These oscillations represent the dynamic behavior of a particular hole in a perforated plate. Their mean amplitude and frequency, averaged over all possible values of v for a plate, may then be taken as characteristic for the whole plate. In Equation (29), R is, in fact, linearized at $t = 0$. For finite values of R, the denominator of the right-hand side of this equation indicates that v is less than $\sqrt{4AC}/B$. This upper bound for v makes it possible to estimate $\bar{R}$ and $\bar{F}$ which may be used to test the validity of the linearized model. Note that while Equation (27) is the formal solution for Equation (25), Equation (28) is proposed as a linearized solution for Equation (25). Thus while Equation (27) satisfies the boundary condition introduced, $v = v_o$ at $t = 0$, Equation (28) reduces to Equation (29) from which the amplitudes may be derived.

For nonstationary values of p, dp/dt not equal to zero, $F(t)$ has finite values, Equation (13). Consider the simple case of $-p = p_o \sin wt$, $F(t) = -\frac{1}{A}\frac{dp}{dt} = \frac{p_o w}{A}\cos wt$.

Equation (12) becomes the differential equation for an oscillatory system with a periodic forcing function, that is

$$\ddot{v} + P\dot{v} + Q^2 v = \frac{p_o w}{A}\cos wt \quad (33)$$

for which the solution is (*4*)

$$v(t) = v + v_s(t) \quad (34)$$

where v = the transient phenomenon, given by Equations (27) and (28), and is the solution of the reduced equation, Equation (25); and $v_s(t)$ = the steady state phenomenon.

$$v_s(t) = \frac{p_o w/A}{\sqrt{(Q^2 - w^2)^2 + P^2w^2}}\cos(wt - \lambda) \quad (35)$$

$$= \frac{p_o w}{\sqrt{(C - Aw^2)^2 + B^2v^2w^2}}\cos(wt - \lambda) \quad (36)$$

and

$$\lambda = \cos^{-1}\left[\frac{(Q^2 - w^2)}{\sqrt{(Q^2 - w^2)^2 + P^2w^2}}\right] \quad (37)$$

$$= \cos^{-1}\left[\frac{(C - Aw^2)}{\sqrt{(C - Aw^2)^2 + B^2v^2w^2}}\right] \quad (38)$$

If $F(t)$ takes the form of a Fourier series, Equation (13), $v_s(t)$ may be obtained by superposition, giving Equation (41), which is a summation of component terms similar to that given by Equations (38) and (36).

In order to obtain an estimate of the oscillatory pressure amplitude in terms of the system parameters A, B, and C, the following assumptions are made: the amplitude of the steady state phenomenon is taken to be $\bar{R}$, the mean characteristic amplitude of flow through the system; and the frequency of the steady state phenomenon equals $\bar{F}$, the mean characteristic frequency, Hence, from Equation (36), with $v = R$

$$p_o = \bar{R}\sqrt{(C - Aw^2)^2 + B^2R^2w^2} \Big/ w \quad (39)$$

where

$$w = 2\pi\bar{F} \quad (40)$$

Thus if $F(t)$ is given by Equation (13), the pressure amplitude will be given by the algebraic sum of component amplitude terms similar to Equation (39).

EXPERIMENTAL OBSERVATIONS

Visual observations of the overall pressure drop across the middle plate of a three-plate 18-in. diameter column described in reference *11* with ½-in. diameter perforations and 2-in. weir height for the air-water system operating at the seal point indicated that this fluctuated with varying amplitude. The amplitude as measured by an inclined gauge manometer connected to shielded pressure tappings was in the range 0.1 to 0.2 in. of water. The frequency was recorded by counting every third fluctuation, with the aid of a stop-clock and a hand-operated counter. The mean frequency of sixteen readings was found to be 2 to 4 cycles/sec.

DISCUSSION

We may use the above analysis to estimate the magnitude of the amplitude and frequency of the pressure fluctuations to be expected at the seal point as: Taking t_p = 1/16 in., $h_L = h_w$ = 2 in., ρ_G = 0.074 lb./ cu. ft., D_h = ½ in., V_c = 3.24 cu.ft. for a column diameter of 18 in. and plate spacing of 2 ft. and c = 1,100 ft./sec., one obtains $A = 9.35$, $B = 8.46 \times 10^4$, and $C = 2.76 \times 10^4$. From Equation (22), for $\bar{v} = 0$, periodic solutions for the dynamic equation exist if

$$0 < \dot{v} \leq \frac{1}{k} < \frac{g}{f} = \frac{C}{B} = 0.326 \text{ cu.ft./sec.}^2$$

or an acceleration of 240 ft./sec.² through the hole. Since the sealing hole velocity for this case is of the order of 10 ft./sec., frequencies of up to 12 cycles/sec. are to be expected.

Since Equation (29) is valid only for finite values of R, we can estimate the corresponding maximum value of v_o:

$$v_o \leq (\sqrt{4AC}/B) = 1.20 \times 10^{-2} \text{ cu.ft./sec.}$$

which is equivalent to an average amplitude

$\bar{R} = (4AC/B^2v_o) = 0.0120$ cu. ft./sec., Equation (30).

The mean frequency given by Equation (32) is

$$\bar{F} = \frac{1}{2\pi}\frac{C}{Bv_o}\sin^{-1}\left(\frac{Bv_o}{\sqrt{4AC}}\right) = \frac{C}{4Bv_o} \leq 6.8 \text{ cycles/sec.},$$

which is comparable to the experimental result.

The amplitude of the oscillatory pressure given by Equation (39) is

$$(12/\rho_L g_c)p_o = (12/\rho_L g_c)\left[\bar{R}\sqrt{(C - Aw^2)^2 + B^2\bar{R}^2w^2} \Big/ w\right] = 0.075 \text{ in.}$$

of water for the case of simple harmonic representation of the pressure $-p = p_o \sin wt$. If p takes the form of a complex Fourier series, the mean amplitude will be the algebraic sum of component pressure amplitudes of this magnitude and may be expected to be of the order of 0.1 in water.

The dynamic equation, Equation (8), was simulated on an analog computer which gave amplitudes of the order of tenths of an inch of water and a frequency of about 5 cycles/sec. for the system.

Equation (31) predicts that the instantaneous frequency is dependent upon $C^{1/2}$, that is, $V_c^{-0.50}$, which may be compared with $V_c^{-0.46}$ in Brown's correlation of experimental data.

CONCLUSION

From the foregoing analysis it appears that even if a very simple input, for example, a constant input, of vapor is introduced at a given plate, at some plate above, the output will be periodic which will cause stable, periodic pressure fluctuations in the space between the plates. In particular, an input given by Equation (13) will lead by the linearized model, Equation (25), to the following output (4):

$$v = \tfrac{1}{2}\bar{p} + \sum_{n=1}^{\infty} p_n^\circ \sin(nwt - \beta_n + \alpha_n) \qquad (41)$$

where

$$p_n^\circ = p_n \Big/ \left\{ \left[1 - \frac{n^2w^2}{Q}\right]^2 + \frac{4Pn^2w^2}{Q} \right\}^{1/2}$$

$$\alpha_n = \tan^{-1}[2Pnw/(PQ - n^2w^2)], \quad (0 < \alpha_n < \pi)$$

A constant or periodic input will therefore give rise to a periodic output, and the analysis carried out here then shows that periodic and stable oscillations will always arise between perforated plates at low loads. This last restriction was introduced to permit the use of a simple expression for the liquid head on a plate, known to be valid at loads corresponding to the seal point. If an acceptable expression becomes available for higher vapor rates, it may be possible to extend the analysis to higher loads and hence account for the oscillations reported and correlated by McAllister and Plank (7). In turn, this may permit a more rigorous study of foam heights, maximum loads, and transfer rates for distillation plates.

At low loads, however, the model provides not only qualitative explanation of the weeping phenomenon, but may be used for quantitative prediction of the seal point.

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NOTATION

A = $4(t_p + h_L)\,\rho_G/\pi D_h^2$, Equation (6a)
A_1 = $(\rho_L/\rho_G)A$
B = $\left(\dfrac{16\rho_G k}{\pi^2 D_h^4 g_c}\right)\left[0.4(1.25-\alpha) + 4f\left(\dfrac{tp}{D_h}\right) + (1-\alpha)^2\right]$, Equation (4a)
C = $c^2\rho_G/V_c$, Equation (2a) = $1/C_o$
C_o = time-averaged capacitance of the space below a plate
c = velocity of sound
c_1, c_2 = constants
D = pitch of perforations on plate
D_h = hole diameter
E = $\tfrac{1}{2}u^2 + G(Z)$
e = ordinate value in Figure 2
F = frequency of pressure fluctuations; $\bar{F}$ = mean frequency
F = functional notation, $F(t)$, Equation (12)
f = functional notation, $f(v)$, Equation (10)
f = Fanning friction factor
G = functional notation, $G(v)$, Equation (18)
g = functional notation, $g(v)$, Equation (11)
g_c = acceleration due to gravity
h_L = static head
h_w = weir height
k = constant
l = vertical distance
M = molecular weight
m = mass of vapor in chamber below the plate
m_G = mass of vapor in hole = $\dfrac{\pi}{4}D_h^2(t_p + h_L)\rho_G$
n = number of holes in plate
n = index number
P = functional notation, $P(t)$, Equation (23)
p = pressure drop across plate
p_i = instantaneous value of p
$\bar{p}$ = mean value of p
p_1 = pressure drop due to capacitance, Equation (2)
p_2 = pressure drop due to resistance, Equation (4)
p_3 = pressure drop due to inertia, Equation (6)
p_4 = pressure drop due to other effects, Equation (7)
p_n = output pressure amplitude, Equation (13)
p_n° = output pressure amplitude, Equation (41)
Q = functional notation, Equation (24)
R = amplitude of fluctuations, Equation (26); R_o value at $t = 0$
T = period
t = time
t_p = plate thickness
u = dv/dt, Equation (15); u_o value at $t = 0$
$\dot{u}$ = du/dt
U_h = average vapor hole velocity
V = molal vapor volume
V_c = volume of space below the plate
v = volumetric vapor flow rate; v_i, $\bar{v}$, instantaneous, mean value
$\dot{v}$ = dv/dt
$\ddot{v}$ = d^2v/dt^2
w = angular velocity, Equations (13) and (41)
z = abscissa value in Figure 2

Greek Letters

α = fractional free area of plate, Equation (3)
α_n = phase shift, Equation (41)
β_n = phase difference, Equation (13)
γ = ratio of specific heats
Γ = limit cycle curve, Figure 2
θ = 2ϕ
τ = period of fluctuation
ϕ = frequency function, Equation (26)
ρ_G, ρ_L = vapor, liquid density

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